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TECHNICAL NOTE

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SOME STUDIES OF LIQUID ROTATION AND VORTEXING IN ROCKET PROPELLANT TANKS

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SOME STUDIES OF LIQUID ROTATION AND VORTEXING IN ROCKET PROPELLANT TANKS

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SUMMARY

This report presents the results of some experimental and theoretical studies of vortex formation while draining fluid from cylindrical tanks, and related liquid dynamic behavior. The experimental studies are largely visual, through motion picture films, but even so reveal many interesting details of vortex formation and behavior, and also include some data on time required for draining with various fluids, tank bottom shapes, and slosh and vortex suppression devices. An analytical study of vortex formation in cylindrical tanks is also presented in an attempt to understand and delineate better the flow mechanisms involved. It is concluded from the present study that while our basic knowledge of vortex formation and behavior is quite meager, baffles and similar devices can easily be provided that will allow the maintenance of adequate flow rates.

INTRODUCTION

Considerable effort has been directed in the past few years towards studies of sloshing in liquid propellant tanks (e.g., refs. 1-5), and much knowledge is now available concerning sloshing characteristics in tanks of various configuration and the effectiveness of various types of suppression devices. Of the various liquid dynamic phenomena, that may occur in rocket tanks, other than normal sloshing, those that involve rotational type motions of the liquid are of special interest because of the torque exerted on the tank through viscous action of the fluid during roll oscillations, changes in inertial distribution, and reduction in flow rate during tank draining as a result of vortex formation Further, previous studies of missile dynamics have largely been based on the quasi-steady assumption that tank draining has no appreciable effect on sloshing force and frequency, although one recent study has shown that there may be significant differences in characteristic fluid responses resulting from high draining rates (ref 6).

It was the purpose of the present study to attempt to gain additional insight and information concerning these problems of liquid rotation, vortexing, and draining. The results obtained are largely qualitative in nature and are based to a large extent on visual and photographic observation; nevertheless, it is believed that the information thereby obtained is of distinct value to rocket designers in providing them with some feeling for the types of liquid dynamic phenomena that occur. A one-reel motion picture supplement to this report is available as a further aid in this regard.*

The studies on which the present report is based are part of a general program of research on liquid dynamics in missile fuel tanks being carried out in the Department of Mechanical Sciences, Southwest Research Institute, under NASA Contract No NASw-146

LABORATORY STUDIES

1 <u>General</u>

Laboratory studies thus far in this program have largely been confined to visual observations of rotating and vortexing flows and some exploratory measurements of rotating flows and the effect of draining on sloshing force response in translation. All tests involve cylindrical tanks of circular cross-section employing the SwRI slosh test facility described in reference 2 These studies will be discussed in more detail below, but in order to provide some orientation for the remarks based on visual observations, a brief description of some of the details of "normal" sloshing will be given first It is interesting to note that some of the details of normal sloshing that will be mentioned are apparently not very well-known

By normal sloshing is usually meant surface wave oscillations primarily in the direction of translational motion, either in- or out-of-phase

*This film is available on loan from NASA Headquarters, Washington 25, D.C. A request card form and a description of the film will be found at the back of this paper, on the page immediately preceding the abstract and index pages.

with the translational motion, depending on the excitation frequency. There is no gross rotation of the liquid about the vertical axis of symmetry of the tank (except as will be specifically discussed below), but there does appear to be some rotational motion set up in each quadrant of the tank cross-section and some interchange of this rotation from one quadrant to another. The rotational motion in each quadrant appears to center about a point just off of the tank axis and close to the tank wall, normal to the direction of translational motion (fig. 1). The general or overall rotational velocity in each quadrant is small compared to the translation frequency except at the point close to the wall and the center line (parallel to the line of motion) of the tank, where the rotational velocity appears to be about the same frequency as that of the translational motion. The rotational frequency decreases as the distance from the center of rotation, in each quadrant, increases.

2. Liquid Rotation

Numerous investigators have noted that during tests with partially filled tanks excited in translation a somewhat unexpected type of liquid rotational motion can be observed (ref. 7). The phenomenon occurs only near resonances of the free surface motions of the liquid and can best be described simply as an apparent "rotation" of the liquid about the vertical axis of symmetry of the tank, superimposed on the normal sloshing motion. The upper and lower limits, on either side of resonance, of the frequency range wherein this type of liquid motion occurs is extremely sensitive to the alignment of the excitation system and therefore the occurrence of this motion can be severely restricted in laboratory tests by careful attention to alignment (ref. 8). More complicated aspects of this motion are, however, present as a type of "beating" also seems to exist: the normal first mode sloshing first begins to transform itself into a rotational motion increasing in angular velocity in (say) the counterclockwise direction, which reaches a maximum and then decreases to essentially zero and then reverses and increases in angular velocity in the clockwise direction, and so on alternately. The frequency of rotation is less than that of the surface wave motion and therefore the liquid appears to undergo a vertical up and down motion as it rotates about the tank axis. The rotational frequency about this up and down axis is about the same as that of the wave motion. The liquid free surface, at least at relatively low values of excitation amplitude, is essentially plane and it is the apparent rotation of this inclined plane about a vertical axis that we are attempting to describe (fig. 2). This phenomenon almost invariably occurs at frequencies in the immediate neighborhood of one of the resonances of the normal sloshing modes, as mentioned above, and occurs whether the fluid has any initial gross rotation or not; the rotational mode can, however, be initiated at any excitation frequency by introducing some disturbance which provides a substantial initial rotation to the liquid.

The phenomenon is obviously quite complex and certainly involves essential nonlinear effects. The motion picture film supplement to this report demonstrates clearly the complexity of this type of liquid motion.

The only attempt to provide an analytical description of this phenomenon appears to be that of reference 7 in which a pendulum analogy leading to the establishment of stability boundaries for this type of motion is proposed. The theory is linear and does not consider the apparent reversal of direction of rotation, but nevertheless does demonstrate the existence of a threshold frequency below which this rotational motion of the liquid due to translational excitation of the tank will not occur. The analysis assumes a two-dimensional stream function for vortex motion, the core of which exhibits rigid body rotation, and the validity of a conical pendulum analogy; however, the strength of the vortex is not predicted. The predicted domain of liquid rotation, near the resonant sloshing frequency, is compared with experiment, with qualitative agreement.

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During the course of these studies involving rotation, at frequencies near translational resonance, measurements were made of the frequency boundaries within which rotation occurred. The resonant frequency was approached slowly from both below and above, at various liquid heights and excitation amplitudes, and the boundary frequency was recorded as that at which rotation was initiated. The resulting data is shown in figure 3, in comparison with that of reference 7. The excitation for this model was provided by a table on v-groove casters riding on somewhat rough inverted steel angles. In previous tests with smoother motion, it was noted that the resonant frequency could be approached somewhat more closely before rotation occurred (ref. 8). More critical comparisons between these analytical results and experiment in terms of velocity and/or pressure distributions appear to pose considerable problems.

It may be noted that while this type of liquid motion is of considerable interest from a scientific viewpoint, it is of considerably less importance in practical application. This is so because of the fact that slosh suppression devices are almost universally employed in actual liquid propellant tanks, and these same slosh suppressors are equally effective in suppressing the liquid rotational motions. In particular, it has been noted in these studies that conical ring baffles (ref. 2), which are very efficient for suppression of normal sloshing, appear to eliminate rotational sloshing completely. Perforated vertical baffles, mounted normal to the direction of the translational motion, decrease the amplitude of normal sloshing by some 50 - 70% and also completely eliminate rotational sloshing; when mounted parallel to the direction of the translational motion, such baffles have no effect on normal sloshing but again completely eliminate rotational sloshing.

3 Effect of Draining on Sloshing Force and Frequency

Analyses of missile and rocket dynamics are based on the quasi-steady assumption that tank draining has no appreciable effect on sloshing force and frequency. In order to determine whether varying liquid level, due to draining, has any significant effect on the force response due to sloshing, several runs were made at various frequencies and excitation amplitudes, while the tank was draining and undergoing translational excitations. The flow rate involved in these tests was determined by gravity and the size of the drain opening at the tank bottom and had an average value of approximately 0.0202 ft^3/sec or, in nondimensional form, $v/d^2\sqrt{\bar{a}d}$ = 0.00226 where v is flow rate, d is tank diameter, and a is longitudinal acceleration. This would correspond to a rate of 2.9 ft^3/sec (1300 gal/min) in a tank of 8.75 ft diameter at 1g or 6.5 ft^3/sec (2900 gal/min) in the same tank at 5g. For each run, a fixed excitation amplitude and frequency were held, and records made as the liquid level passed through heights at which data was available for fixed levels. Frequencies near the first resonant frequency were chosen, as any effect should be more pronounced in this region. At one of the frequencies, the liquid passed through resonance as the liquid height decreased from h/d = 1.00 to zero.

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The general results of these tests show, at least for the flow rate given above, the effect of varying liquid level due to draining is not significant and that sloshing force response data from tests at discrete (fixed) liquid levels can be used with complete confidence to represent the response in a draining tank as the liquid surface passes through these levels (quasi-steady analysis). The contrary results of reference 6 correspond to much higher draining rates.

4 Vortexing

As a tank is drained, a vortex is often observed to form as the fluid level drops to relatively low values. The vortex or rotational flow surrounds a hollow core (figure 4) and consequently results in marked decrease in flow rate by reducing the effective cross-sectional area of the drain outlet. Even in the case where draining is accomplished by virtue of a pump, rather than free draining under gravity (or some given axial thrust, which amounts to the same thing), the flow rate is greatly affected and pump efficiency deteriorates because of the entrained gas and vapor.

It is therefore of some interest to understand draining behavior as an aid to developing effective techniques for the suppression of vortex formation.

Some qualitative visual studies of vortexing have been made employing a transparent plexiglas model tank of approximately 11-inch inside diameter and a 1-1/8-inch diameter drain hole. The following flow conditions have been observed visually, employing both flat and conical bottoms on the model tank: (a) draining by gravity, with and without initial liquid rotation, and (b) draining by pumping, with and without initial liquid rotation. Observations were also made for each draining condition both with and without a "cross"-type baffle (fig. 5) over the drain orifice, and the general influence of viscosity was observed by employing water at various temperatures and various water-sugar solutions. These observations were largely conducted with the drain orifice centered in the tank bottom, but later additional tests were conducted with orifice off-set at one-half of the tank radius. Some observations of vortex formation were also made while the tank was subjected to translational excitation to induce sloshing. Droplets of coloring matter were introduced into the liquid as an aid to flow visualization.

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While the motion picture film which forms a supplement to this report is without question the only effective means by which one can really begin to understand some of the interesting and important aspects of the vortexing problem, it is possible to discuss here certain of those aspects and to draw some general conclusions of more immediate practicality to the missile designer.

The observations of draining without sloshing were carried out with the liquid initially at a depth equal to the tank diameter and for various initial states of motion. The measured drain time was that required to drain the tank to within 3/8" of being empty (flat bottom model) and was markedly affected only by whether or not a strong initial rotational motion of the liquid was present; the onset of vortexing was markedly affected by even a small amount of initial liquid rotation and, of course, by the presence of a baffle over the drain orifice. The following case descriptions were evolved from numerous individual observations:

(a). Tank filled and draining initiated immediately. No baffle device. Vortexing appeared when the fluid level dropped to approximately 3/4" from bottom. Drain time: 24.0 sec.

(b) Tank filled and draining initiated after substantial time delay. No baffle device. Some vortexing appeared when the fluid level dropped to approximately 3/4" from bottom. Drain time: 23.0 sec.

(c). Tank filled and draining initiated immediately. Cross type perforated baffle above drain hole. No vortexing. Drain time: 22.5 sec.

(d). Tank filled and strong initial liquid rotation introduced artificially. No baffle device. Vortexing appeared when the fluid level dropped to approximately 3-1/2" from bottom. Drain time: 40 sec.

(e) Tank filled and strong initial liquid rotation introduced artificially. Cross type baffle. No vortexing. Drain time: 23.0 sec.

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Observations were also made for the case of draining while undergoing translational excitation to induce sloshing (drain times were not recorded):

(f). Draining while undergoing normal sloshing appears much as in case (a) above.

(g). Draining with liquid rotation (very near sloshing first mode resonance) again appears similar to case (d) above. The time for draining is very long.

(h). Draining while undergoing normal sloshing with the cross type baffle is similar to case (c) above except that the drain time for the last 1" of fluid depth is considerably greater.

Some general conclusions that may be drawn from these studies of vortexing are as follows:

A. The effects of small amounts of initial liquid rotation (and earth rotation) on vortex formation appear to be negligible for both gravity and pump draining. In fact, for the configurations employed in these studies, there was no vortex formation at all without considerable initial rotation, except near the very end of the draining period.

B. The range of viscosity provided in these studies by various combinations of temperature and percentage sugar added to the water, had no apparent effect on vortex formation or strength.

C. Some benefit is gained by locating the drain orifice away from the tank center line. The vortex tends to form initially by a depression or "crater" in the free surface near or at the center with a long thin filament or core extending down to the orifice; when the orifice is off-center, this filament tends to be curved and unstable so that what vortex action does occur is only intermittent in nature.

D. Sloshing tends to break up a vortex as soon as it is formed, and thus acts relatively effectively as a vortex suppressor.

E. The liquid rotation phenomenon discussed earlier in this report, and which occurs only near a slosh resonance frequency, is potentially the most important agent for the early production of strong vortexing, as demonstrated by case (g) above. However, customary baffling arrangements employed to suppress normal sloshing are also quite effective in suppressing liquid rotation and hence will ameliorate this factor. Also, a simple cross-type baffle over the orifice effectively eliminated the effect of vortexing on the flow rate through the orifice; even when a vortex formed above the baffle, the baffle was effective in breaking up the vortex and preventing it from extending into the orifice.

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THEORETICAL STUDIES OF VORTEX FORMATION

1. Background

As a further attempt to understand better the basic mechanisms involved in vortex formation in draining tanks, certain theoretical analyses are presented in the present section of this report. As will be obvious to the reader, the present analyses do not constitute anything like a complete or satisfactory theory; and yet it is believed that they have value in demonstrating certain aspects of vortex phenomena. No detailed comparison with experimental data is yet possible, but a certain degree of qualitative agreement, based on the visual observations discussed earlier, seems to be evident.

The analyses which follow are restricted to a tank free from any external translational or rotational excitation, but which rotates with the earth. The effect of rigid body rotation of the tank on the liquid has been studied previously in reference 9. We begin with a brief review of previously published theoretical work.

In reference 10, a two-dimensional line vortex is assumed to exist initially and diffuse up to a certain small time, then the flow is assumed to be practically nonviscous. The latter is rotational with an assumed radial velocity which is independent of axial distance and vanishes at infinity The axial flow is determined from the continuity equation which requires, strictly speaking, a special bottom of the container. Two of the momentum equations are approximately satisfied, provided that the inertia terms are negligible. The third equation, including inertial terms, is then used to determine the growth of the tangential velocity component and then the growth of the free surface, which seems to be qualitatively correct. In this paper a basic irrotational flow will be considered without an initial line vortex.

In reference 11, a simplified model is constructed consisting of a perforated rotating cylinder through which water is constantly added at the draining rate. The axial velocity is assumed to be small, so that the basic radial flow is approximately that of a two-dimensional sink flow. When the turbulent viscous effect is taken into account in the growth of the tangential velocity component, the predicted free surfaces can be made to agree with experiments. In this paper, the circular tank is at rest and no liquid is added during draining.

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In both reference 12 and the present paper, the effect of earth rotation is considered. In reference 12, an infinite domain is considered. A two-dimensional sink flow is assumed for the outer domain in which the tangential velocity grows linearly with time. For the inner domain, a radial flow proportional to r accompanied by a downward flow proportional to the axial distance is assumed. The corresponding angular velocity is found to grow exponentially with time. The tangential velocity is then matched through a transition domain. Some numerical estimation of the angular velocity at later times was made in reference 12 which seems to justify further exploration.

In reference 13, a one-dimensional theory including friction is employed to predict the flow rate through a tank. In the present paper, the draining rate will be prescribed to predict the three-dimensional flow inside the tank.

2. Effect of Earth Rotation on a Draining Tank

Assume the tank is axial symmetric and fixed on earth without excitation. Referring to tank fixed cylindrical coordinates (fig. 6) and neglecting the centrifugal forces, the basic equations of momentum and continuity are:

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} = \frac{u_\theta^2}{r} - \frac{\partial p}{\rho \partial r} + 2(u_z \sin\psi_T \sin\theta + u_\theta \cos\psi_T)\Omega \quad (1)$$

$$\frac{\partial u_\theta}{\partial t} + u_z \frac{\partial u_\theta}{\partial z} + u_\theta \frac{\partial u_\theta}{r \partial \theta} + u_r \frac{\partial u_\theta}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{r \partial \theta} - \frac{u_r u_\theta}{r} - 2(u_r \cos\psi_T - u_z \sin\psi_T \cos\theta)\Omega \quad (2)$$

$$\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + u_\theta \frac{\partial u_z}{r \partial \theta} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g - 2(u_\theta \sin\psi_T \cos\theta + u_r \sin\psi_T \sin\theta)\Omega \quad (3)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{\partial u_\theta}{r \partial \theta} + \frac{\partial u_z}{\partial z} = 0 \quad (4)$$

The effect of viscosity, which tends to diffuse the vortex core and retard its growth, has been neglected in the above equations in order to estimate an upper bound for the rate of growth of vortexing.

Let

$$p = \tilde{p} - 2u_z \sin\psi_T \sin\theta \cdot \Omega \quad (5)$$

to absorb the last term in equation (2), additional terms proportional to Ω being introduced in equations (1) and (3) by employing $\tilde{p}$. Ω is approximately 7.272×10^{-5} rad/sec = 2π rad/day. The terms containing Ω tend to destroy axial symmetry of flow through a symmetric tank. However, unless the tank is very large and the draining is very slow, one may reasonably assume the deviation from axial symmetry is small; that is, we shall neglect the terms proportional to Ω in the r and z components of the momentum equation and assume $\frac{\partial}{\partial \theta} = 0$ in all the equations. Writing p for $\tilde{p}$ hereafter, we have

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} = \frac{u_\theta^2}{r} - \frac{\partial p}{\rho \partial r} \quad (6)$$

$$\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \tag{7}$$

$$\frac{\partial (ru_\theta)}{\partial t} + u_z \frac{\partial (ru_\theta)}{\partial z} + u_r \frac{\partial (ru_\theta)}{\partial r} = -2u_r \cos\psi_T \cdot \Omega \tag{8}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r) + \frac{\partial u_z}{\partial z} = 0 \tag{9}$$

We shall assume that if $\frac{u_\theta^2}{r}$ remains negligible, in particular with respect to $u_r \frac{\partial u_r}{\partial r}$ or u_θ deviates very slightly from a distribution depending solely on r, then the flow is practically irrotational. In the first instance, $\frac{u_\theta^2}{r}$ is completely neglected in equation (6), while in the second case the main part can be absorbed in the distribution of pressure p and the remaining part neglected. We shall consider mainly the case in which the flow is initially at rest. The growth of u_θ will be determined until $\frac{u_\theta^2}{r}$ contradicts the assumption discussed.

Let the basic irrotational flow be described by

$$\left\{\begin{array}{ll} \dfrac{\partial \phi}{\partial r} = u_r = -\dfrac{1}{r}\dfrac{\partial \psi}{\partial z} & (10a) \\[2ex] \dfrac{\partial \phi}{\partial z} = u_z = \dfrac{1}{r}\dfrac{\partial \psi}{\partial r} & (10b) \end{array}\right. \qquad \left\{\begin{array}{llll} u_r = w_o(t)\bar{u}_r & (11a) & u_z = w_o(t)\bar{u}_z & (11b) \\[2ex] \psi = w_o(t)\bar{\psi} & (11c) & & \end{array}\right.$$

The characteristic equation corresponding to equation (8) is

$$\frac{dt}{1} = \frac{dz}{u_z} = \frac{dr}{u_r} = \frac{d(ru_\theta)}{-2u_r\Omega\cos\psi_T} \tag{12}$$

from which

$$ru_\theta + 2\Omega r \cos\psi_T = \text{constant} \tag{13}$$

$$\bar{\psi}(r, z) = \text{constant} = \bar{\psi} \tag{14}$$

Equation (14) implies that

$$z = z(r, \bar{\psi}) \text{ or } r = r(z, \bar{\psi}) \tag{15}$$

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Also, from equation (12) we could have either

$$\int_o^t w_o dt - \int^z \frac{dz}{[u_z(r, z)]_{r = r(z, \bar{\psi})}} = \text{constant} \tag{16a}$$

or

$$\int_o^t w_o dt - \int^r \frac{dr}{[u_r(r, z)]_{z = z(r, \bar{\psi})}} = \text{constant} \tag{16b}$$

The solution of equation (8) is then

$$ru_\theta + 2\Omega r \cos\psi_T = F\left[\bar{\psi}(r, z), \int_o^t w_o dt - \int^z \frac{dz}{[u_z(r, z)]_{r = r(z, \bar{\psi})}}\right] \tag{17a}$$

or

$$ru_\theta + 2\Omega r \cos\psi_T = F\left[\bar{\psi}(r, z), \int_o^t w_o dt - \int^r \frac{dr}{[u_r(r, z)]_{z = z(r, \bar{\psi})}}\right] \tag{17b}$$

Subject to the initial condition, at t = 0,

$$u_\theta = \Omega_r r \tag{18}$$

where $\Omega_r = 0$, if initially at rest.

In general, it is not always possible to express the second integrals in equation (16a) or (16b) explicitly in terms of elementary functions. In order to obtain some estimate of the growth of vortexing, approximate formulas applicable to specified domain, say $r \longrightarrow 0$, will be sought, since in such cases $\bar{\psi}$ can be approximated by one or two terms and the final solution can be obtained in terms of known functions. The following cases will be discussed.

A. <u>Case 1</u> - Growth of vortexing around a two-dimensional sink.

The basic flow is a two-dimensional sink flow:

$$\phi = -\frac{m}{2\pi} \ln r \quad , \quad u_r = -\frac{m}{2\pi r} \quad , \quad u_z = 0$$

$$\psi = \frac{m}{2\pi} z \quad , \quad m = m(t)$$

The growth of u_θ is given by

$$r u_\theta = -r^2 \Omega \cos\psi_T + \Omega \cos\psi_T \left(\int_0^t m dt + \pi r^2 \right) \frac{1}{\pi}$$

$$+ \Omega_r \left(\int_0^t m dt + \pi r^2 \right) \frac{1}{\pi} \tag{19}$$

Thus for $\Omega_r = 0$, u_θ behaves as $\frac{1}{r}$ as $r \longrightarrow 0$, and the strength of u_θ for m = constant increases linearly with time. For $\Omega_r \neq 0$, the additional vortex increases also linearly with time.

B. <u>Case 2</u> - Growth of vortexing around a three-dimensional sink.

The basic irrotational flow is given by (ref. 14) as

$$\phi = \frac{m}{4\pi\sqrt{r^2 + z^2}} \qquad\qquad \psi = \frac{m}{4\pi}\,\frac{z}{\sqrt{r^2 + z^2}}$$

where m is twice the total flux absorbed by the sink for a semi-infinite domain.

The tangential velocity u_θ is found to be

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$$ru_\theta = -r^2\Omega\cos\psi_T + \Omega\cos\psi_T\left\{\frac{3}{4\pi}\,\frac{r^3}{(r^2+z^2)^{3/2}}\int_0^t m(t)dt + r^3\right\}^{2/3}$$

$$+\Omega r\left\{\frac{3}{4\pi}\,\frac{r^3}{(r^2+z^2)^{3/2}}\int_0^t m(t)dt + r^3\right\}^{2/3} \tag{20}$$

The growth is approximately linear for m = constant if r & z = 0(L) but

$$\frac{1}{L^3}\int_0^t m(t)dt \text{ small.}$$

C. <u>Case 3</u> - Flow through a circular drainage; semi-infinite domain.

The irrotational flow through a circular drainage with constant velocity distribution is given (ref. 15) by

$$\phi = \int_0^\infty \left\{e^{-kz} J_0(kr)kdk \int_0^\infty f(r')\,J_0(kr')\,r'dr'\right\} \tag{21}$$

where

$$f(r) = -\left.\frac{\partial\phi}{\partial z}\right|_{z=0} = \begin{cases} w_o \text{ for } r' < a \\ 0 \text{ for } r' > a \end{cases} \tag{21a}$$

or, let $V = \pi a^2 w_o$

$$\phi = \frac{V(t)}{\pi a} \int_o^\infty e^{-kz} J_o(kr)\, J_1(ka)\, \frac{dk}{k} \tag{22}$$

$$\psi = -\frac{rV(t)}{\pi a} \int_o^\infty e^{-kz} J_1(kr)\, J_1(ka)\, \frac{dk}{k} \tag{23}$$

Since

$$\int_o^\infty e^{-kz} J_o(kr) dk = \frac{1}{\sqrt{r^2 + z^2}}$$

and

$$\int_o^\infty e^{-kz} J_1(kr) dk = \frac{r}{\sqrt{r^2 + z^2}\left(z + \sqrt{r^2 + z^2}\right)}$$

for

$$r \longrightarrow 0, \quad z > 0,$$

$$\frac{\partial \phi}{\partial z} \cong \frac{-V(t)}{a\pi} \cdot \frac{a}{\sqrt{a^2 + z^2}\left(z + \sqrt{a^2 + z^2}\right)} \tag{24}$$

$$\psi \cong \frac{-r^2 V(t)}{a\pi} \cdot \frac{a}{\sqrt{a^2 + z^2}\left(z + \sqrt{a^2 + z^2}\right)} \tag{25}$$

corresponding to equation (16a) along characteristics

$$\frac{1}{\pi} \int_0^t V dt + \frac{1}{3}(a^2 + z^2)^{3/2} - \frac{1}{3} a^3 + a^2 z + \frac{1}{3} z^3 = \text{constant} \tag{26}$$

Further approximation is made for $z \gg a$ or $z \ll a$ by retaining only the dominant terms.

It is found that for $z \gg a$ $(a \gg r)$

$$r u_\theta \cong - r^2 \Omega \cos \psi_T + (\Omega \cos \psi_T + \Omega_r) r^2 \left\{ \frac{3}{2\pi z^3} \int_0^t V dt + 1 \right\}^{2/3} \tag{27}$$

which is in agreement with the result in Case 2 as $m = 2V$ and that for $0 < z \ll a$ $(a \gg r \to 0)$

$$ru_\theta \cong \Omega_r r^2 + r^2(\Omega \cos \psi_T + \Omega_r)\,\frac{\frac{1}{\pi}\int_0^t V\,dt}{a^3\left(1+\frac{z}{a}\right)} \qquad (28)$$

By comparison of equations (27) and (28), it shows that the growth of tangential velocity is much faster at the center of the bottom than at the center far away from the bottom.

Suppose equation (28) is applied to a large circular tank of radius ten times the radius of the drain; $\frac{a}{R} = 0.1$. With an initial liquid depth of 2R, the maximum amplifying factor near the end of draining (at $z = 0$) is

$$\frac{\frac{1}{\pi}\int_0^t V\,dt}{a^3} \to \left[\frac{1}{\pi}\,\frac{R^3}{a^3}\cdot\frac{1}{R^3}\cdot \pi R^2 \cdot 2R\right] = 2,000$$

If

$$\Omega_r = 0, \qquad \cos\psi_T = 1 \qquad (\cos\psi_T \cong 1/2 \text{ in Texas})$$

$$\left(\frac{u_\theta}{r}\right)_{max} \doteq 2,000 \times 7.27 \times 10^{-5} \text{ rad/sec} = .0145 \text{ rad/sec} = .02314$$

cyl/sec, which is quite slow.

Next, if $a = \frac{1}{24}$ ft, $V_L = 5\ \text{ft}^3$, then the maximum amplifying factor $\frac{1}{\pi}\,\frac{V_L}{a^3} = 2.20 \times 10^4$. Let $\cos\psi_T = \frac{1}{2}$, then $\left(\frac{u_\theta}{r}\right)_{max} = 0.800$ rad/sec = 0.1272 cyl/sec, which is slow but easily observable. If the initial velocity $u_\theta = 0$ $(\Omega_r = 0)$, the vortexing should grow in the direction of axial component of earth rotation which is counterclockwise viewed from the top of the tank in the northern hemisphere and clockwise in the southern hemisphere.

D <u>Case 4</u> - Growth of vortexing of flow through a circular tank

The velocity potential and stream function will be given later. In order to obtain some simple estimates, we shall assume $r \to 0$, z sufficiently large, say $z \geq \frac{1}{2}R$, and use one term of the infinite series, i.e.,

$$\bar{\Psi} \cong \frac{1}{2}\bar{B}_o r^2 - r\bar{B}_1 e^{-\frac{k_1 z}{R}} J_1\left(k_1 \frac{r}{R}\right) \tag{29}$$

$$-\bar{u}_r \cong \frac{k_1}{R}\bar{B}_1 e^{-\frac{zk_1}{R}} J_1\left(k_1 \frac{r}{R}\right)$$

where

$$\bar{\Psi} = \frac{\psi}{w_o} \quad , \quad \bar{B}_n = \frac{B_n}{w_o}$$

D
1
2
1
2

$$B_n = \frac{-2R\left(\frac{a}{R}\right)}{k_n^2 J_o^2(k_n)} J_1\left(k_n \frac{a}{R}\right) \cdot (-w_o) \qquad n = 1,\ 2,\ \cdots \tag{30a}$$

$$B_o = -w_o \frac{a^2}{R^2} \tag{30b}$$

Equation (16b) yields, along characteristics,

$$\int_o^t w_o dt + \frac{R}{2k_1}\left(\frac{1}{\frac{1}{2}\bar{B}_o}\right) \ln\left[\frac{\frac{1}{2}B_o^2 r^2 - \bar{\Psi}}{-\bar{\Psi}}\right] \cong \text{constant} \tag{31}$$

The tangential velocity is then ($r \to 0$, z sufficiently large)

$$ru_\theta \cong (\Omega\cos\psi_T + \Omega_r) \cdot 4R^2\left(\frac{r}{a}\right) \frac{e^{-\frac{k_1 z}{R}} J_1\left(k_1\frac{a}{R}\right)}{k_1^2 J_o^2(k_1)} J_1\left(k_1\frac{r}{R}\right)\left\{1 - e^{-\frac{a^2}{R^2}\left[\frac{k_1}{R}\int_o^t w_o dt\right]}\right\}$$
$$+ r^2\Omega_r \cos\psi_T \tag{32}$$

For t small and w_o = constant, $\frac{u_\theta}{r}$ grows linearly with time For t large, $\frac{u_\theta}{r}$ approaches constant at fixed location r and z. Unfortunately, for both z and r small, the series solution converges very slowly. Therefore, one cannot obtain the effect of finite tank radius on the growth of vortexing near the center of the tank bottom.

If $\frac{r}{R}$, $\frac{a}{R} \to 0$, $t \to \infty$, $\pi w_o a^2 = V$ = constant, $\Omega_r = 0$ and $z = 0$,

equation (32) yields

$$\left(\frac{u_\theta}{r}\right)_{max} = \frac{4\Omega \cos \psi_T}{J_o^2(k_1)} \qquad \text{where} \qquad \frac{4}{J_o^2(k_1)} \cong 24.6$$

For $\cos \psi_T = \frac{1}{2}$, $\left(\frac{u_\theta}{r}\right)_{max}$ takes the value 1.428×10^{-4} cyl/sec. If this is the order of magnitude of the growth near the bottom, the vortexing is negligible, unless there is initial rotation considerably larger than that due to earth rotation.

When a/R is very small, equation (28) should yield better order of magnitude estimates near the center of the bottom than does equation (32). Preliminary experiments were conducted using a tank of radius 1/2 ft with a drain of about 1-1/8 in diameter and filled to heights = R or 2R. When the initial rotation in the tank was carefully damped out with a perforated screen, no vortexing was directly observable throughout the draining period. This is consistent with the upper bound estimated by equation (28).

3. Irrotational Flow Through Finite Circular Tank With Centrally Located Circular Drainage (Flat Bottom)

We shall assume the effect of earth rotation is negligible and the flow is axisymmetric. Let $\Omega_r = 0$ and the tank be stationary. By neglecting the effect of viscosity, the flow can be further assumed to be irrotational. The velocity potential must satisfy the following equation

$$\frac{\partial^2 \phi}{\partial z^2} + \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\cancelto{0}{\frac{\partial^2 \phi}{\partial \theta^2}} = 0 \qquad (33)$$

with the boundary conditions that $\frac{\partial \phi}{\partial r} = 0$ at $r = R$

and

$$\left.\frac{\partial\phi}{\partial z}\right|_{z=0} = \begin{cases} 0 & r > a \\ w_o(t) & r < a \end{cases}$$

Two approximate approaches to obtain solutions will be considered, depending on a remaining boundary condition.

A. Case 1 - Assume the flow behaves as if the tank is filled to infinity where $\left.\frac{\partial\phi}{\partial z}\right|_{z\to\infty} = -w_o \frac{a^2}{R^2}$

By the method of separation of variables and (modified) Dini series expansion (ref. 16), it is found that

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$$\phi = \sum_{n=1}^{\infty} B_n e^{-zk_n/R} J_o\left(k_n \frac{r}{R}\right) + B_o z = w_o(t)\bar{\phi}(r, z) \tag{34}$$

where

$$B_o = -w_o \frac{a^2}{R^2}, \quad B_n = \frac{-2R\left(\frac{a}{R}\right)}{k_n^2 J_n^2(k_n)} J_1\left(k_n \frac{a}{R}\right)(-w_o) \tag{30}$$

$$k_1 = 3.8317, \quad k_2 = 7.0156, \quad k_3 = 10.1735, \text{ etc. (ref. 10)}$$

The corresponding stream function is

$$\psi = -\sum_{n=1}^{\infty} rB_n e^{-k_n z/R} J_1\left(k_n \frac{r}{R}\right) + \frac{1}{2} B_o r^2 \tag{35}$$

By integration of the momentum equation, the pressure is given by [cf. eq. (39)]

$$\frac{p}{\rho} = \frac{p_F(t)}{\rho} - \frac{\partial\phi}{\partial t} - \frac{1}{2}(u_r^2 + u_z^2) - gz + gh(t) + \frac{\partial w_o}{\partial t} \cdot \frac{2}{R^2} \int_0^R (\bar{\phi})_{z=h+\zeta}\, r dr$$
$$+ \frac{2}{R^2} \int_0^R \frac{1}{2}(u_r^2 + u_z^2)_{z=h+\zeta}\, r dr \tag{36}$$

where

$$h(t) = h_o - \int_o^t w_o dt \cdot \frac{a^2}{R^2}$$

Since for slow draining $\frac{\partial w_o}{\partial t}$ is small and for constant flow draining $w_o \cong$ const. , the term $\frac{\partial w_o}{\partial t}$ can be neglected. On the free surface $z = h(t) + \zeta(r, t)$, $\rho = p_F$, equation (36) determines the shape of the free surface As a first approximation, which should be good for $\frac{\zeta}{R}$ small and $\frac{\partial w_o}{\partial t}$ small, we shall calculate ζ by

$$\frac{\zeta}{R} = \frac{w_o^2}{gR}\left\{-\frac{1}{2}\,(\bar{u}_r^2 + \bar{u}_z^2)_{z \cong h} + \frac{2}{R^2}\int_o^R \left[\frac{1}{2}\,(\bar{u}_r^2 + \bar{u}_z^2)\right]_{z \cong h} r dr\right\} \tag{37}$$

where

$$\bar{u}_r = u_r/w_o, \ \bar{u}_z = u_z/w_o$$

$$\frac{1}{R^2}\int_o^R \left[(\bar{u}_z^2)\right]_{z \cong h} r dr = \sum_{n=1}^{\infty} \frac{2\left(\frac{a}{R}\right)^2}{k_n^2 J_o^2(k_n)}\, J_1^2\left(k_n \frac{a}{R}\right) e^{-2h\left(\frac{k_n}{R}\right)} + \frac{1}{2}\,\frac{a^4}{R^4} \tag{38a}$$

$$\frac{1}{R^2}\int_o^R \left[\bar{u}_r^2\right]_{z \cong h} r dr = \sum_{n=1}^{\infty} \frac{2\left(\frac{a}{R}\right)^2}{k_n^2 J_o^2(k_n)}\, J_1^2\left(k_n \frac{a}{R}\right) e^{-2h\left(\frac{k_n}{R}\right)} \tag{38b}$$

The example shown in figure 7 was computed by taking eleven terms of the series. The center of the free surface descends a little faster than at the tank wall. For the case $\frac{\partial w_o}{\partial t} \cong 0$, $\frac{w_o^2}{2gR} = \frac{h}{R}$, the free surface is still practically flat down to $\frac{h}{R} = \frac{1}{8}$. For the case $\frac{w_o^2}{gR} = 4.3$* the discharging rate is faster than the first case and the free surface forms a crater near the center, which should be observable in a tank of 6" radius. The motion picture films mentioned earlier in this report were studies carefully and measurements of the free surface form were made for a number of selected frames; this experimental data is also shown in figure 7 and indicates qualitative agreement with the theory. Besides the small difference in average depth, the remaining discrepancy is due mainly to the following two factors: (a) the irrotational flow is viewed in the theory as though it

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*The theoretical analysis was actually carried out for a flow rate of 4.0; this slight difference should not greatly affect the comparison.

comes from infinity and this may not be an adequate description of the flow field at a later stage, and (b) the initial rotation of the liquid in the experiment is small, but not negligible, and therefore vortexing may actually have altered the free surface shape.

B. Case 2 - Impose the linearized pressure condition on the free surface

The unsteady Bernoulli's equation can be written as

$$\frac{p}{\rho} = \frac{p_F}{\rho} - \left\{ \frac{\partial \phi}{\partial t} + \frac{1}{2} q^2 + gz - gh - \frac{2}{R^2} \int_0^R \left(\frac{\partial \phi}{\partial t} \right)_{z=h+\zeta} r\,dr - \frac{1}{R^2} \int_0^R (q^2)_{z=h+\zeta}\, r\,dr \right\} \tag{39}$$

Assume the free surface is nearly flat and the square of the velocity negligible. Let ϕ absorb an arbitrary function of time such that

$$p = -\rho g(z-h) - \rho \frac{\partial \phi}{\partial t} + p_F, \quad \text{on } z = h + \zeta \cong h(t) \tag{40}$$

Since $\frac{Dp}{Dt} = 0$, $\frac{Dz}{Dt} = \frac{\partial \phi}{\partial z}$, $\frac{Dh}{Dt} = \frac{-w_o a^2}{R}$ on the free surface and

$$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} + g \frac{w_o(t) a^2}{R^2} \cong 0 \tag{41}$$

Let

$$\phi = \sum_{n=1}^{\infty} \left[A_n \cosh \frac{k_n z}{R} - B_n \sinh \frac{k_n z}{R} \right] J_o\left(k_n \frac{r}{R} \right) + B_o z + \int_0^t \int_0^t \frac{\ddot{w}_o a^2}{R^2} h(t)\,dt\,dt + (-\dot{B}_o h_o t) - B_o h_o \tag{42}$$

where

$$B_o = -\frac{w_o(t)a^2}{R^2}$$

Equation (41) yields

$$\left\{ \sum_{n=1}^{\infty} \left[\ddot{A}_n \cosh \frac{k_n z}{R} - \ddot{B}_n \sinh \frac{k_n z}{R} \right] J_o\left(k_n \frac{r}{R}\right) \cancelto{\cong 0}{- \frac{\ddot{w}_o a^2}{R} z + \frac{\ddot{w}_o a^2}{R^2} h(t)} \right.$$

$$+ \sum_{n=1}^{\infty} \left[g \frac{k_n}{R} A_n \sinh \frac{k_n z}{R} - g \frac{k_n}{R} B_n \cosh \frac{k_n z}{R} \right] J_o\left(k_n \frac{r}{R}\right)$$

$$\left. + \cancelto{0}{g\left(-\frac{w_o a^2}{R^2}\right) + g\left(\frac{w_o a^2}{R^2}\right)} \right\}_{z \cong h} \cong 0$$

Therefore, it is required that

$$\ddot{A}_n + g\left(\frac{k_n}{R}\right)\left(\tanh \frac{k_n h}{R}\right) A_n = \tanh\left(\frac{k_n h}{R}\right) \ddot{B}_n + \left(g \frac{k_n}{R}\right) B_n \qquad (43)$$

Since at $t = 0$, $\phi = 0$, $h = h_o$, $p_{\text{at } z = h_o} = p_F$; thus equation (40) implies $\left.\frac{\partial \phi}{\partial t}\right|_{t=0} = 0$. Equation (42) yields

$$A_n(0) = B_n(0) \tanh \frac{k_n h_o}{R} \qquad (44)$$

Partial differentiation of equation (42) yields

$$\dot{A}_n(0) = \dot{B}_n(0) \tanh \frac{k_n h_o}{R} \qquad (45)$$

The exact solution of equation (43) subject to the conditions (44) and (45) is difficult to obtain. However, if $\tanh\left(\frac{k_n h}{R}\right) \cong 1$ is applied in all these equations, then $A_n(t) \cong B_n(t)$ is the approximate solution. If $h \geq \frac{R}{2}$, $\tanh\frac{k_n h}{R} \geq \tanh\frac{k_1}{2} \cong 0.957$. For smaller h, the error is larger. Within this approximation

$$\phi \cong \sum_{n=1}^{\infty} B_n e^{-\frac{k_n z}{R}} J_o\left(k_n \frac{r}{R}\right) + B_o z + \int_o^t \int_o^t \frac{\ddot{w}_o a^2}{R^2} h(t)dtdt - \dot{B}_o h_o t - B_o h_o \qquad (46)$$

The flow field (velocity) at any time is independent of the last three terms, and hence is essentially the same as the previous case.

CONCLUSIONS

Problems of liquid rotation and vortexing in liquid rocket propellant tanks are intrinsically of great complexity and scientific interest. Quantitative experimental data of sufficient accuracy and detail to lend real insight and understanding into the governing fluid mechanisms are extremely difficult to acquire, however, and theoretical advances are hindered by this same lack of insight and understanding and by the dominant role obviously played by the nonlinear character of the liquid motion. Nevertheless, it is believed that the qualitative studies and related theory, presented in this report and its motion picture supplement, go far in assisting the missile or rocket designer in realizing the type of flow conditions with which he should be cognizant.

Fortunately, it appears that adequate baffling, or other suppression devices, can be provided relatively easily on an empirical basis to ameliorate these particular types of liquid behavior; especially since such suppressors, which are almost invariably provided for normal sloshing anyhow, are also effective in regard to liquid rotation.

Southwest Research Institute,
San Antonio, Texas, November 29, 1961.

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APPENDIX - NOTATION

a	radius of the circular drainage
$\bar{a}$	axial acceleration
d	tank diameter
g	gravitational acceleration
$h(t)$	average liquid height at time t
k_n	nth root of $J_1(k_n) = 0$
m	strength of a sink
p	pressure
$\tilde{p}$	approximate pressure
p_F	pressure on the free surface
q	$\sqrt{u_z^2 + u_r^2}$
r, θ, z	cylindrical coordinates (fig. 6)
R	radius of the circular tank
t	time
u_r, u_θ, u_z	radial, tangential and axial velocity components, respectively
v	flow rate
V	total flux through the drainage
V_L	total volume of liquid before draining
w_o	axial velocity at center of drainage, $(z = 0, r = 0)$
X_o	excitation amplitude

D
1
2
1
2

APPENDIX - NOTATION (Cont'd)

$\zeta(r, t)$	free surface location measured from the mean liquid height
ρ	density of the liquid
ϕ	velocity potential
ψ	stream function
ψ_T	latitude between tank axis and north pole (fig. 6)
Ω	angular velocity of earth rotation about its own axis
Ω_r	initial rigid body rotation about the tank axis
ω	exitation frequency

D
1
2
1
2

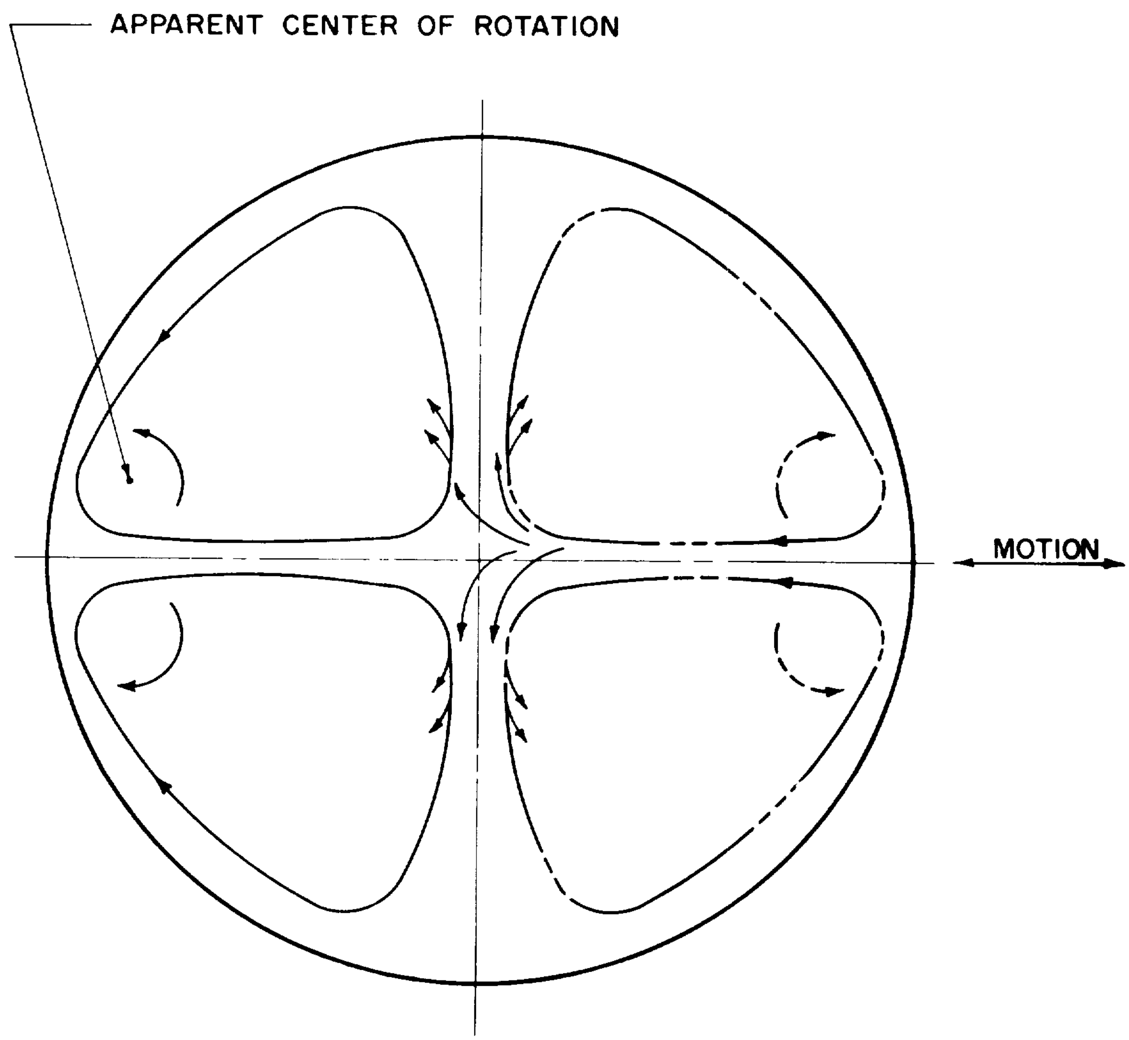

Figure 1.- Sketch showing interchange of fluid and rotation within quadrants during normal sloshing.

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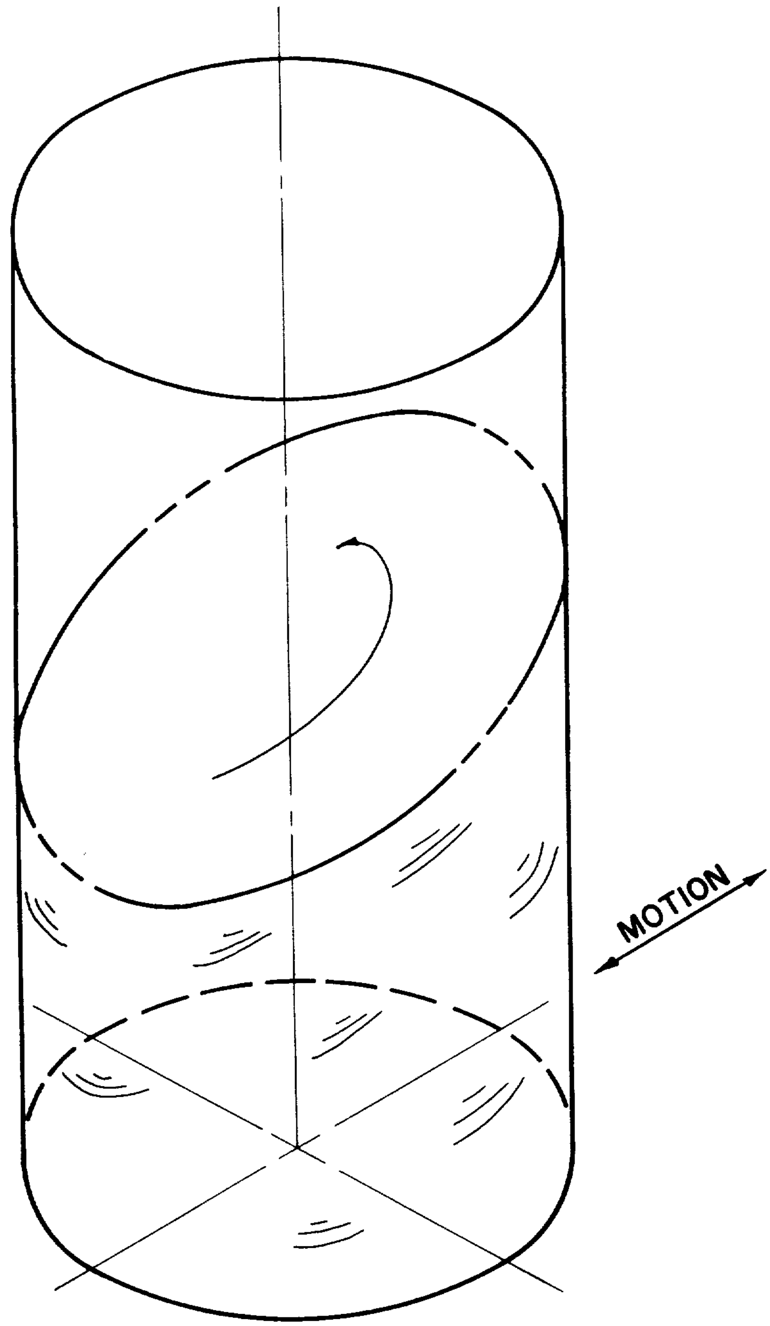

Figure 2.- Sketch showing rotational liquid motion.

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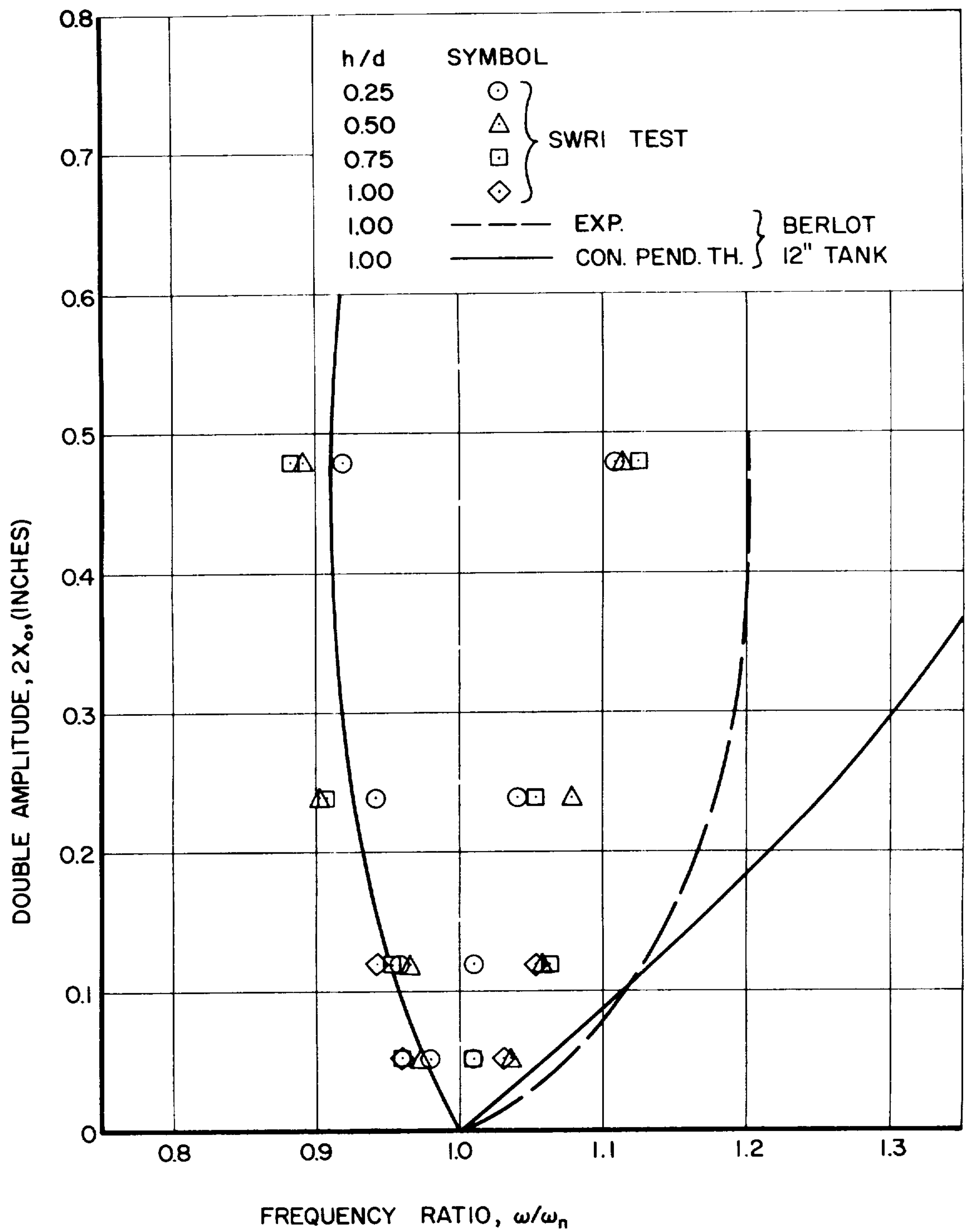

Figure 3.- Comparison of experimental data with theoretical stability boundaries for rotational motion.

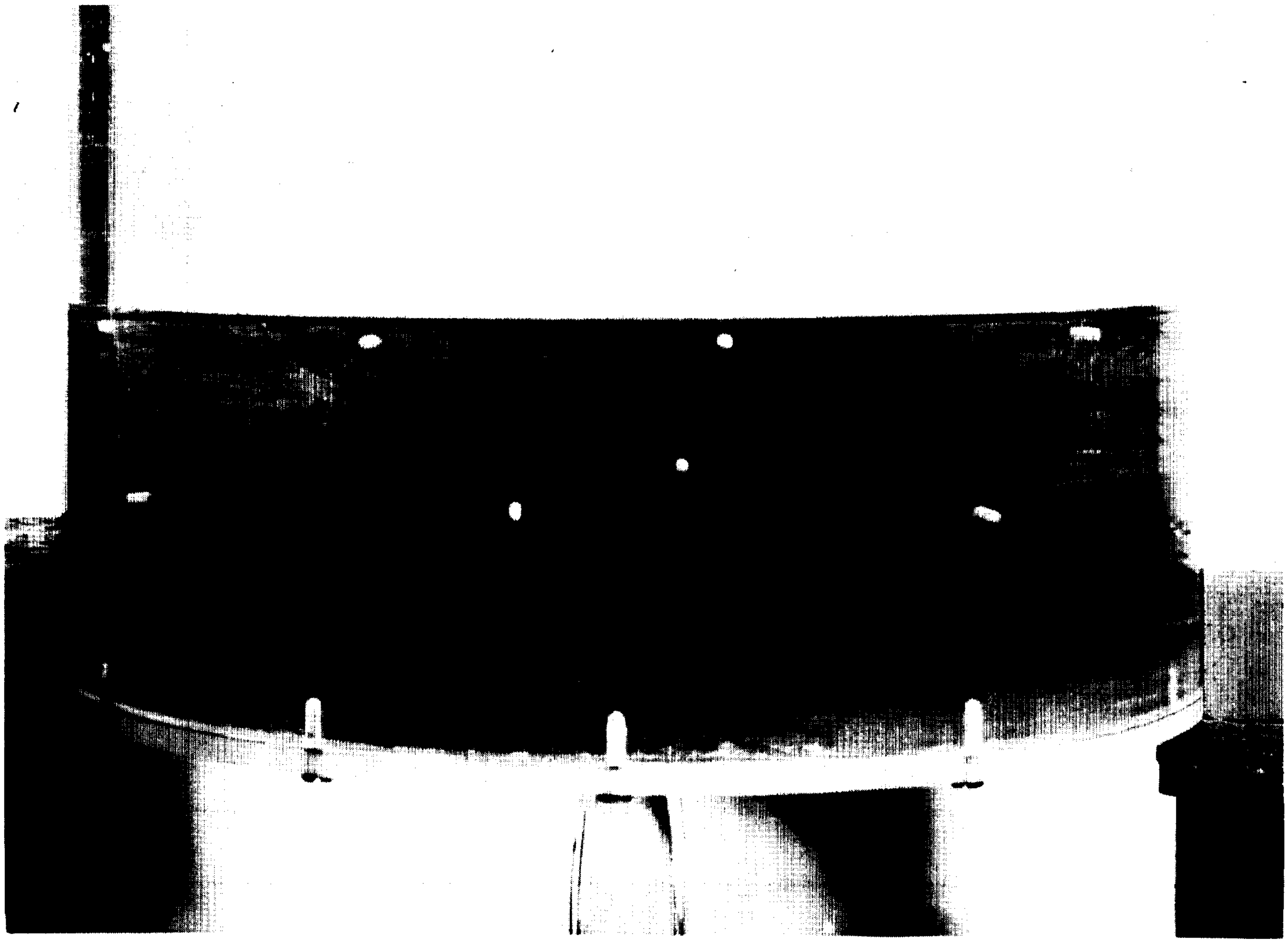

Figure 4.- Vortex formed during steady gravity draining from a cylindrical tank. L-62-1

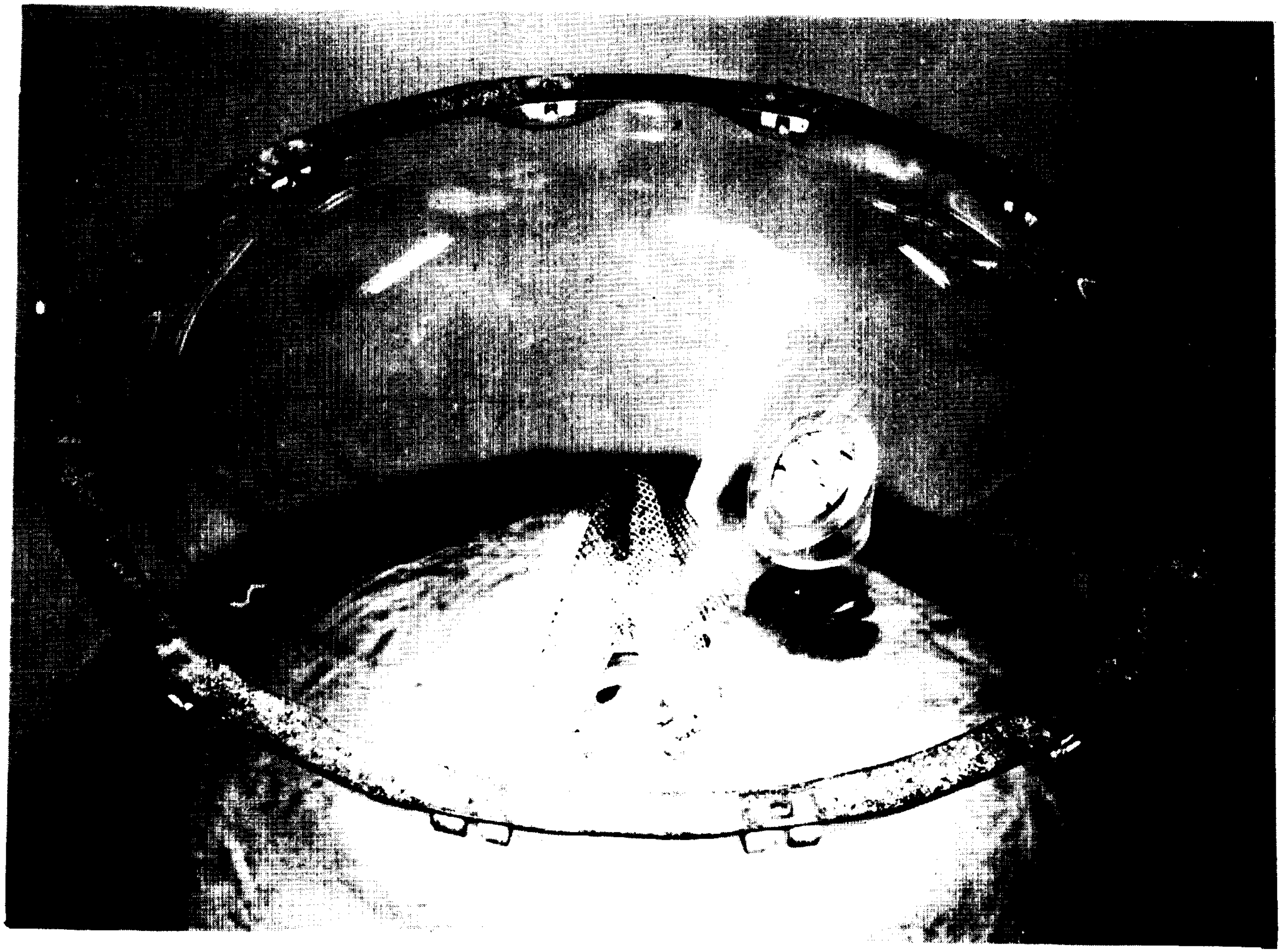

Figure 5.- Cross type of vortex baffle. L-62-2

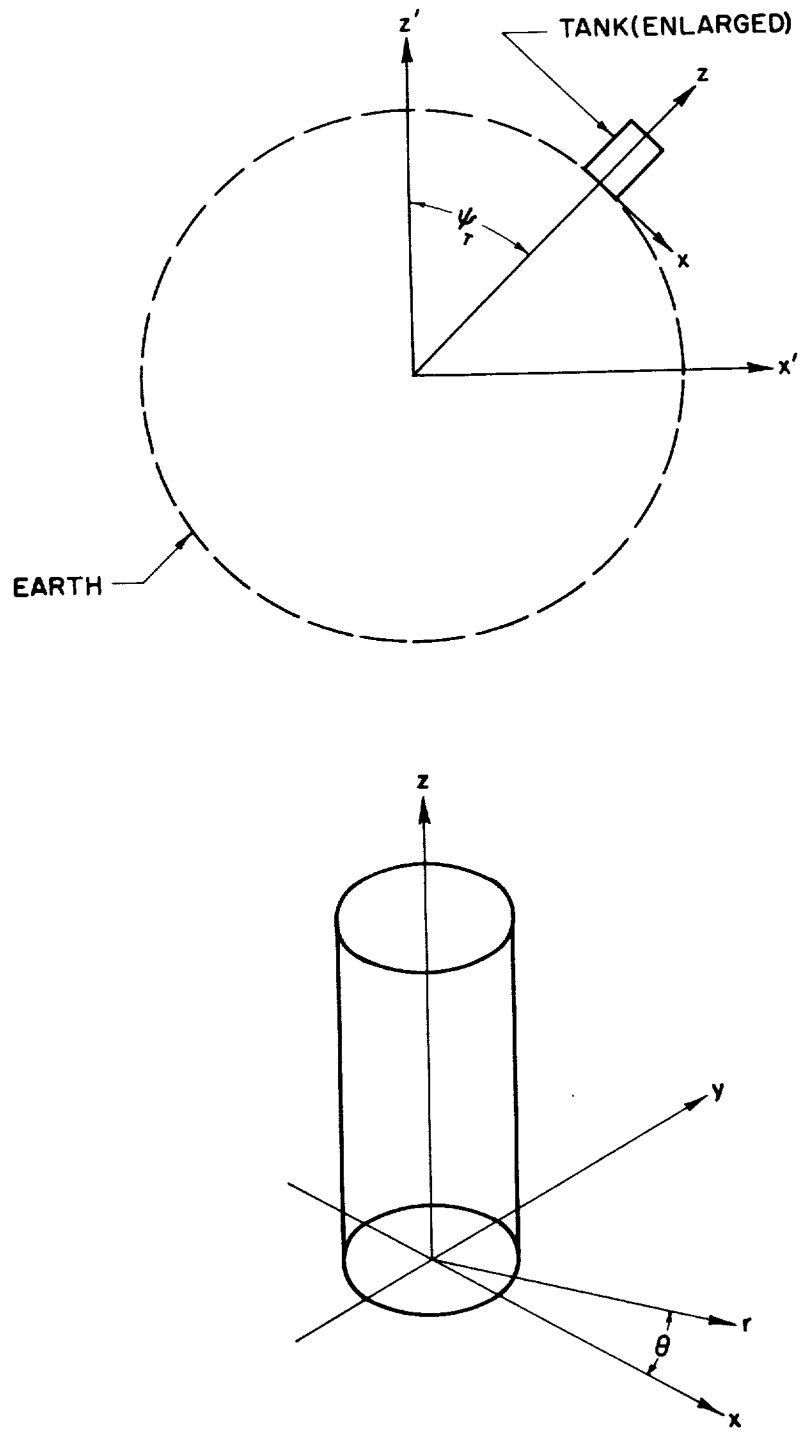

Figure 6.- Coordinate system for theoretical analysis of vortex formation.

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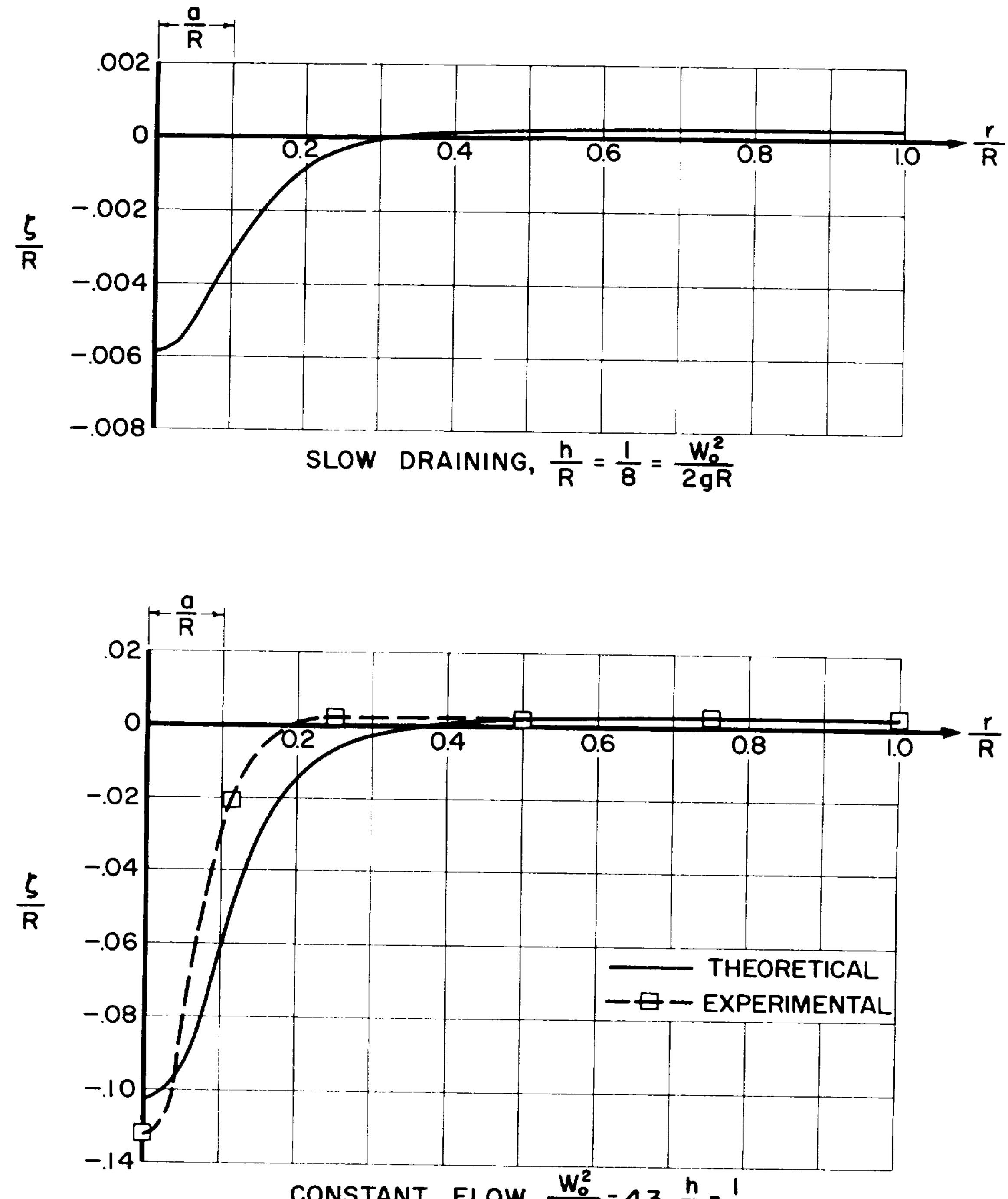

Figure 7.- Liquid free surface shape for irrotational draining.